**Death by Uber**

**Stable Matchings of Blue Effects to Red Targets**

**Math 340 Group Project Materials**

**Spring 2021**

**Step 1: Learn to identify stable matchings**

* **Task**: Watch the following introductory video on the matching problem: [https://www.youtube.com/watch ?v=RE5PmdGNgj0](https://www.youtube.com/watch%20?v=RE5PmdGNgj0)
* For our project we will consider sets of **BLUE** effects (think munitions, jamming, or inserting malicious code) combined with **RED** targets (physical things like buildings or ships, as well as virtual things like networks)
  + Obviously each **BLUE** effect will have preferences for which target it is most suited to impact. Possible ways to prioritize targets from the effect perspective could be:
    - Range: How easily can I reach the target?
    - Timeliness: How likely am I to reach the target before it moves?
    - Survivability: How likely am I to survive to the target?
    - Lethality: How likely am I to create the desired effect on the target?
  + In a similar way, each **RED** target will have preferences for which effect it is most suited to be serviced by. Possible ways to prioritize effects from the target’s perspective could be:
    - Desirability of Effect: How well suited is the effect to my mission?
    - Target Value: How large is the impact on **RED** versus the cost of the effect?
    - Proportionality: How many unintended consequences are expected from the effect?
  + Clearly neither list is complete. For our purposes, we will always be given the preferences, and won’t worry too much about how they were developed.
* **Exercise**: Below, there is a set of **BLUE** effects, **RED** targets, and a preference list for each element. There are also two matchings. Determine whether each matching is stable. If it is not stable, identify a rogue couple in the matching.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **BLUE** Effects | Preferences  (most preferred…least preferred) | | | | | |  | **RED** Targets | Preferences  (most preferred…least preferred) | | | | | |
| **B1** | R6 | R1 | R5 | R3 | R4 | R2 |  | **R1** | B1 | B4 | B6 | B3 | B2 | B5 |
| **B2** | R5 | R6 | R2 | R3 | R4 | R1 |  | **R2** | B3 | B1 | B6 | B4 | B5 | B2 |
| **B3** | R6 | R5 | R1 | R4 | R3 | R2 |  | **R3** | B6 | B3 | B5 | B1 | B2 | B4 |
| **B4** | R6 | R1 | R5 | R4 | R3 | R2 |  | **R4** | B6 | B3 | B1 | B4 | B2 | B5 |
| **B5** | R2 | R3 | R6 | R5 | R1 | R4 |  | **R5** | B5 | B2 | B4 | B1 | B6 | B3 |
| **B6** | R6 | R3 | R4 | R5 | R1 | R2 |  | **R6** | B1 | B4 | B6 | B2 | B3 | B5 |

1. Is a stable matching? If not, write down a rogue couple.

(B3, R4) is not stable.

1. Is a stable matching? If not, write down a rogue couple.

(B4, R1), (B5, R3) are rogue couples. The matching is not stable.

1. In our context, what does a “rogue couple” represent in terms of target selection? How important are stable matchings in this context? What do stable matchings provide to the mission?

A rogue couple represents when an effect is not matched with its preferred target and that target is not matched with its preferred effect. Stable matchings are important because we do not want a case where munitions are being used for something that a cyberattack would be most suited for. Stable matchings provide a more effective and efficient mission.

**Group Members**: Taylor Coffey, Josiah Stearns, Daniel Perry

**Step 2: Learn to build stable matchings**

* **Task**: Watch the following video on building stable matchings: <https://www.youtube.com/watch?v=6vgHIImFwHo>
* **Exercise**: Below, there is a set of **BLUE** effects, **RED** targets, and a preference list for each element. Build a stable matching from this information.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **BLUE** Effects | Preferences  (most preferred…least preferred) | | | | | | | | | |
| **B1** | R7 | R2 | R8 | R6 | R9 | R1 | R4 | R3 | R5 | R10 |
| **B2** | R5 | R2 | R4 | R3 | R1 | R8 | R9 | R10 | R7 | R6 |
| **B3** | R5 | R10 | R7 | R4 | R3 | R1 | R8 | R9 | R6 | R2 |
| **B4** | R2 | R7 | R9 | R6 | R3 | R10 | R8 | R5 | R1 | R4 |
| **B5** | R3 | R7 | R1 | R10 | R9 | R5 | R6 | R2 | R8 | R4 |
| **B6** | R10 | R1 | R4 | R5 | R9 | R6 | R3 | R2 | R8 | R7 |
| **B7** | R4 | R9 | R5 | R8 | R2 | R1 | R7 | R3 | R6 | R10 |
| **B8** | R8 | R5 | R9 | R7 | R4 | R2 | R1 | R3 | R10 | R6 |
| **B9** | R10 | R4 | R8 | R5 | R6 | R2 | R1 | R7 | R9 | R3 |
| **B10** | R2 | R4 | R1 | R9 | R6 | R3 | R10 | R8 | R5 | R7 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **RED** Targets | Preferences  (most preferred…least preferred) | | | | | | | | | |
| **R1** | B7 | B4 | B1 | B2 | B3 | B6 | B8 | B9 | B10 | B5 |
| **R2** | B6 | B1 | B5 | B9 | B8 | B10 | B2 | B4 | B3 | B7 |
| **R3** | B8 | B5 | B7 | B10 | B4 | B6 | B2 | B9 | B1 | B3 |
| **R4** | B5 | B6 | B3 | B2 | B10 | B9 | B4 | B1 | B8 | B7 |
| **R5** | B2 | B1 | B8 | B7 | B9 | B10 | B4 | B6 | B3 | B5 |
| **R6** | B1 | B10 | B4 | B7 | B3 | B5 | B2 | B9 | B6 | B8 |
| **R7** | B4 | B5 | B8 | B1 | B7 | B9 | B3 | B10 | B2 | B6 |
| **R8** | B2 | B7 | B5 | B8 | B4 | B10 | B1 | B6 | B9 | B3 |
| **R9** | B5 | B9 | B7 | B4 | B1 | B8 | B3 | B10 | B6 | B2 |
| **R10** | B3 | B1 | B6 | B2 | B8 | B9 | B7 | B4 | B10 | B5 |

Stable Matching:

**Group Members**: \_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Step 3: Learn to optimize stable matchings**

* **Task**: Watch the following video on optimizing stable matchings: <https://www.youtube.com/watch?v=n4KKgKpp--0>
* **Exercise**: Below, there is a set of **BLUE** effects, **RED** targets, and a preference list for each element. Build a stable matching from this information that maximizes **BLUE** effect preferences. Build another stable matching from this information that maximizes **RED** target preferences.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **BLUE** Effects | Preferences  (most preferred…least preferred) | | | | | | | | | |
| **B1** | R7 | R2 | R8 | R6 | R9 | R1 | R4 | R3 | R5 | R10 |
| **B2** | R5 | R2 | R4 | R3 | R1 | R8 | R9 | R10 | R7 | R6 |
| **B3** | R5 | R10 | R7 | R4 | R3 | R1 | R8 | R9 | R6 | R2 |
| **B4** | R2 | R7 | R9 | R6 | R3 | R10 | R8 | R5 | R1 | R4 |
| **B5** | R3 | R7 | R1 | R10 | R9 | R5 | R6 | R2 | R8 | R4 |
| **B6** | R10 | R1 | R4 | R5 | R9 | R6 | R3 | R2 | R8 | R7 |
| **B7** | R4 | R9 | R5 | R8 | R2 | R1 | R7 | R3 | R6 | R10 |
| **B8** | R8 | R5 | R9 | R7 | R4 | R2 | R1 | R3 | R10 | R6 |
| **B9** | R10 | R4 | R8 | R5 | R6 | R2 | R1 | R7 | R9 | R3 |
| **B10** | R2 | R4 | R1 | R9 | R6 | R3 | R10 | R8 | R5 | R7 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **RED** Targets | Preferences  (most preferred…least preferred) | | | | | | | | | |
| **R1** | B7 | B4 | B1 | B2 | B3 | B6 | B8 | B9 | B10 | B5 |
| **R2** | B6 | B1 | B5 | B9 | B8 | B10 | B2 | B4 | B3 | B7 |
| **R3** | B8 | B5 | B7 | B10 | B4 | B6 | B2 | B9 | B1 | B3 |
| **R4** | B5 | B6 | B3 | B2 | B10 | B9 | B4 | B1 | B8 | B7 |
| **R5** | B2 | B1 | B8 | B7 | B9 | B10 | B4 | B6 | B3 | B5 |
| **R6** | B1 | B10 | B4 | B7 | B3 | B5 | B2 | B9 | B6 | B8 |
| **R7** | B4 | B5 | B8 | B1 | B7 | B9 | B3 | B10 | B2 | B6 |
| **R8** | B2 | B7 | B5 | B8 | B4 | B10 | B1 | B6 | B9 | B3 |
| **R9** | B5 | B9 | B7 | B4 | B1 | B8 | B3 | B10 | B6 | B2 |
| **R10** | B3 | B1 | B6 | B2 | B8 | B9 | B7 | B4 | B10 | B5 |

1. Stable matching that maximizes **BLUE** effect preferences:
2. Stable matching that maximizes **RED** target preferences:
3. What are the pros and cons of each matching? If you were the **BLUE** commander, which matching would you prefer? Why?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Group Members**: \_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Step 4: Apply insights to matchings on incomplete bipartite graphs**

* **Task**: Watch the following video on matchings on bipartite graphs: <https://www.youtube.com/watch?v=HZLKDC9OSaQ>
* So far we’ve been under a couple of faulty assumptions:
  + We’ve assumed there are the same number of **BLUE** effects as there are **RED** targets. Unfortunately, sometimes we are resource-constrained and we have more **RED** targets than **BLUE** effects; and, sometimes we’re in an intel-constrained environment where we have the opposite problem.
  + We’ve also assumed every **BLUE** effect can be matched with any **RED** target. This is not always the case. A **BLUE** effect might not have the range to reach a target, the lethality to successfully engage a target, or it might not be the type of effect we want to create on the target.
* We need a development in our model. Consider the following bipartite graph:

* The **BLUE** vertices on the left represent the 15 **BLUE** effects, and the **RED** vertices on the right represent the 13 **RED** targets (so we are in an intel-constrained environment).
* An edge from a **BLUE** effect to a **RED** target means that **BLUE** effect can be matched to that **RED** target.
* Each edge from a **BLUE** effect needs to be prioritized with the other edges from that **BLUE** effect. Similarly, each edge into a **RED** target needs to be prioritized with the other edges into that **RED** target. The following tables give these preferences:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **BLUE** Effects | Preferences  (most preferred…least preferred) | | | | | |  | **RED** Targets | Preferences  (most preferred…least preferred) | | | | | |
| **B1** | R3 | R13 | R9 | R2 | R11 |  |  | **R1** | B14 | B10 | B2 | B4 |  |  |
| **B2** | R12 | R7 | R4 | R10 | R1 |  |  | **R2** | B3 | B13 | B12 | B1 |  |  |
| **B3** | R2 | R5 | R7 | R11 |  |  |  | **R3** | B1 | B9 | B15 | B6 |  |  |
| **B4** | R10 | R13 | R1 | R8 |  |  |  | **R4** | B2 | B14 | B8 | B5 |  |  |
| **B5** | R12 | R11 | R4 | R8 |  |  |  | **R5** | B3 | B9 | B12 | B7 |  |  |
| **B6** | R9 | R3 | R6 |  |  |  |  | **R6** | B6 | B15 | B10 | B8 |  |  |
| **B7** | R8 | R12 | R5 |  |  |  |  | **R7** | B14 | B3 | B9 | B2 |  |  |
| **B8** | R4 | R6 | R10 | R12 |  |  |  | **R8** | B4 | B5 | B15 | B11 | B7 |  |
| **B9** | R3 | R7 | R5 |  |  |  |  | **R9** | B6 | B1 | B10 |  |  |  |
| **B10** | R6 | R9 | R1 |  |  |  |  | **R10** | B4 | B2 | B13 | B8 |  |  |
| **B11** | R12 | R8 | R11 |  |  |  |  | **R11** | B11 | B15 | B1 | B5 | B3 |  |
| **B12** | R2 | R5 | R13 |  |  |  |  | **R12** | B7 | B8 | B11 | B2 | B5 |  |
| **B13** | R10 | R2 | R13 |  |  |  |  | **R13** | B13 | B12 | B4 | B1 |  |  |
| **B14** | R4 | R7 | R1 |  |  |  |  |  |  |  |  |  |  |  |
| **B15** | R6 | R11 | R8 | R3 |  |  |  |  |  |  |  |  |  |  |

* **Exercise**: Determine if there is a stable matching that covers all of the **RED** targets. If there is a stable matching that covers all the **RED** targets, build the one that maximizes either the **BLUE** effect preferences or the **RED** target preferences based on how you answered the exercise question in **Step 3**.

1. Is there a stable matching that covers all of the **RED** targets? You need to argue for your position using **Hall’s Theorem**.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Stable matching that maximizes **BLUE** effect (or **RED** target) preferences:
2. Which two **BLUE** effects have not been used yet? If you were the **BLUE** commander, how would you use the remaining two **BLUE** effects?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Group Members**: \_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Step 5: Report your findings**

* **Task**: Build a project report describing your methodology, results, and recommendations. This should be a **stand-alone** product. It is the only product that will be **graded for content**. Your report should have the following sections:
  + **Abstract** 
    - 1-2 paragraphs that describe the project and your major results (keep it short!)
  + **Section 1: Stable Matchings** (max of 2 pages)
    - Briefly describe the initial setup of the problem (boys, girls, preferences, matchings, rogue couples, etc)
    - Describe an algorithm to find a stable matching (if you have code or pseudo code you can just insert it here)
    - Describe how your algorithm optimizes one group above the other. Include how you choose which group is optimized.
    - Describe changes to your algorithm when the preferences lists are incomplete (make sure to include a brief discussion on existence of a matching in this case)
    - No need to describe the application (effects and targets) here, just stick to the theory
  + **Section 2: Applying stable matchings to the BLUE effect to RED target problem** (max of 3 pages)
    - Briefly describe the setup of the application (reinterpret Section 1 in light of **BLUE** effects and **RED** targets)
    - Briefly describe your results from the Step 1 exercise (make sure to include answers to all the questions)
    - Briefly describe your results from the Step 2 exercise (make sure to include answers to all the questions)
    - Briefly describe your results from the Step 3 exercise (make sure to include answers to all the questions)
    - Briefly describe your results from the Step 4 exercise (make sure to include answers to all the questions)
  + **Section 3: Documentation Statement & Works Cited**
    - **Your integrity is the most important part of this project.** Do not steal someone else’s intellectual property or jeopardize your integrity.
    - Make sure to cite your sources in accordance with the *Dean’s Academic Honor Memorandum* and *Academic Honor Principles and Guidance*. Citations help your credibility, they don’t undermine it—use them!
    - You are **not required to cite the videos assigned for this project or the course textbook**. However, you can choose to cite either of these in particular instances if you think a citation will save writing space or bolster your argument.